Excitation and conversion of electromagnetic waves in pulsar magnetospheres

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(Received 8 November 2007; published 11 March 2008)

We demonstrate that nonlinear decay of obliquely propagating Langmuir waves into Langmuir and Alfvén waves ($L \to L' + A$) is possible in a one-dimensional, highly relativistic, streaming pair plasma. Such a plasma may be in the magnetospheres of pulsars. It is shown that the characteristic frequency of generated Alfvén waves is much less than the frequency of Langmuir waves and may be consistent with the observational data on the radio emission of pulsars.

DOI: 10.1103/PhysRevE.77.037402 PACS number(s): 52.35.−g, 52.25.Xz

I. INTRODUCTION

It is generally believed that a coherent (brightness temperatures up to $\sim 10^{30}$ K) radio emission of pulsars is generated by plasma instabilities in the one-dimensional, relativistic plasma consisting of electron-positron pairs (e.g., [1]). Such pairs may be produced by γ rays via electromagnetic cascades in the vicinity of the magnetic poles of pulsars that are identified with strongly magnetized ($\sim 10^{9}-10^{13}$ G), rotation-powered neutron stars. Created particles very rapidly lose the momentum component transverse to the magnetic field and flow away along the open magnetic field lines with Lorenz factors ranging from $\gamma_{\min} \sim 10$ to $\gamma_{\max} \sim 10^3$. A two-stream instability was proposed as a mechanism of radio emission of pulsars soon after their discovery [2]. Later, using the available models of pulsar magnetospheres it was argued that if the plasma outflow is stationary, the two-stream instability does not have enough time to be developed before the plasma escapes the pulsar magnetosphere (for a review, see [3]). It was argued [4,5] that the process of pair creation near the pulsar surface is strongly nonstationary and that the pair plasma gathers into separate clouds spaced by $l \sim 10^6$ cm along the direction of its outflow. In this case, at the distance $r_i \simeq 2l\gamma_{\min}^2 \sim 10^8$ cm from the neutron star, fast ($\gamma \simeq \gamma_{\max}$) particles of one plasma cloud overtake slow ($\gamma \simeq \gamma_{\min}$) particles of the preceding cloud and mutual overlapping of the clouds begins. In the overlapping region, there are, in fact, two steams of slow and fast particles, and the condition for the development of two-stream instability is created [6]. For typical parameters of pair plasma in the pulsar magnetospheres the growth rate of the instability is quite sufficient for its development (see below).

The frequency of Langmuir (L) waves generated at the distance r_i from the neutron star is a few 10 times higher than the typical frequency of pulsar radio emission (see [7] and below). Therefore, the wave frequency has to decrease significantly because of some processes to be compatible with the observational data. In this paper, we discuss the process of conversion of L waves into *low-frequency* Alfvén (A) waves that can solve the high-frequency problem.

II. BASIC EQUATIONS

We consider pair plasma in the external uniform magnetic field $\mathbf{B}_0$ directed along the z axis. The behavior of this plasma may be described by the Vlasov and Maxwell equations

$$\frac{\partial}{\partial t}f_\alpha + v_z\frac{\partial}{\partial z}f_\alpha + e_\alpha\left(E_z + \frac{1}{c}[\mathbf{v}\times\mathbf{B}]_z\right)\frac{\partial}{\partial p_z}f_\alpha = 0, \quad (1)$$

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}, \quad \nabla\times\mathbf{E} - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B} = 0, \quad (2)$$

where f_α is the distribution function for the particles of type α, which is normalized so that $\int f_\alpha dp_z = n_\alpha$, $n_0 = \Sigma_\alpha n_\alpha$ is the particle density, $\mathbf{B} \equiv \mathbf{B}_0 + \tilde{\mathbf{B}}$, $\tilde{\mathbf{B}}$ is the magnetic field perturbation, and $\mathbf{j} = \Sigma_\alpha e_\alpha \int \mathbf{v} f_\alpha dp_z$ is the current density.

Our study of nonlinear phenomena will be performed in the frame of the weak turbulence theory. The particle distribution functions and the current density may be written in the perturbation manner

$$f_\alpha = f_\alpha^{(0)} + f_\alpha^{(1)} + f_\alpha^{(2)} + \cdots, \quad \mathbf{j} = \mathbf{j}^{(0)} + \mathbf{j}^{(1)} + \mathbf{j}^{(2)} + \cdots. \quad (3)$$

Here the unperturbed distribution function $f_\alpha^{(0)}$ is assumed to be stationary and homogenous.

Besides, we use the infinite magnetic field approximation (for finite magnetic field effects, see [8]). In this approximation, the distribution function f_α is one dimensional in the velocity space ($\mathbf{v} \parallel \mathbf{B}_0$) and depends only on v_z. Technically, this means that only the z component of $\mathbf{j}^{(i)}$ is nonzero,

$$j_z^{(i)} = \sum_\alpha e_\alpha \int v_z f_\alpha^{(i)} dp_z. \quad (4)$$

In this case from Eqs. (1) and (3) we have

$$\frac{\partial}{\partial t}f_\alpha^{(0)} + e_\alpha\left\langle E_z\frac{\partial}{\partial p_z}f_\alpha^{(1)}\right\rangle = 0, \quad (5)$$

$$\frac{\partial}{\partial t}f_\alpha^{(i)} + v_z\frac{\partial}{\partial z}f_\alpha^{(i)} + e_\alpha\left\langle E_z\frac{\partial}{\partial p_z}f_\alpha^{(i-1)}\right\rangle = 0, \quad (6)$$

where the angular brackets denote an average over one pulsation.

Performing Fourier transformation with respect to temporal and spatial variables, Eq. (6) for $i=1,2$ yields

$$f_\alpha^{(1)}(\bar{k}) = -ie_\alpha \frac{E_z(\bar{k})\,\partial f_\alpha^{(0)}/\partial p_z}{\omega - k_z v_z}, \tag{7}$$

$$f_\alpha^{(2)}(\bar{k}) = \frac{-e^2}{\omega - k_z v_z}\frac{\partial}{\partial p_z}\int d\bar{k}'\ E_z(\bar{k}-\bar{k}')E_z(\bar{k}')\frac{\partial f_\alpha^{(0)}/\partial p_z}{\omega' - k_z' v_z}, \tag{8}$$

where $\bar{k} \equiv (\mathbf{k},\omega)$. The first- ($f_\alpha^{(1)}$) and second- ($f_\alpha^{(2)}$) order perturbations describe the linear dynamics of plasma and three-wave resonant processes, respectively.

Without loss of generality we assume that the wave vector $\mathbf{k}$ lies in the x,z plane. From Eqs. (2), (4), and (7) it can be shown that there are the following three fundamental modes of pair plasma (e.g., [9]). The first one is the extraordinary wave with the electric field perpendicular to the $\mathbf{k},\mathbf{B}_0$ plane. The other two modes are L waves generated in the development of the two-steam instability and A waves into which L waves may be converted (see below). For both these modes their electric fields lie in the $\mathbf{k},\mathbf{B}_0$ plane.

In the infinite magnetic field approximation when only one component (ε_{zz}) of the linear permittivity tensor is nonzero [9],

$$\varepsilon_{zz} = 1 - \sum_\alpha \omega_p^2 \int \frac{F_\alpha}{\gamma^3(\omega - k_z v_z)^2} dp_z, \tag{9}$$

for rather low wave numbers

$$k_z^2 c^2 \ll 2\omega_p^2 \langle \gamma^{-3}\rangle, \quad |k_z k_x| c^2 \ll 2\omega_p^2 \langle \gamma^{-3}\rangle, \tag{10}$$

the dispersion relations can be written in the form

$$\omega_L^2 = 2\omega_p^2\langle\gamma^{-3}\rangle + 3k_z^2c^2\left[1 - \frac{\langle\gamma^{-3}\rangle}{\langle\gamma^{-5}\rangle}\right] + k_x^2c^2\frac{k_z^2c^2}{2\omega_p^2\langle\gamma^{-3}\rangle} \tag{11}$$

for quasilongitudinal L waves and

$$\omega_A^2 = k_z^2c^2\left[1 - \frac{k_x^2c^2}{2\omega_p^2\langle\gamma(1+v_z/c)^2\rangle}\right] \tag{12}$$

for quasitransversal A waves, where $F_\alpha = f_\alpha^{(0)}/n_\alpha$, so that $\int F_\alpha dp_z = 1$, $\omega_p = (4\pi e^2 n_0/m_e)^{1/2}$ is the plasma frequency, and $\langle\cdots\rangle \equiv \int \cdots F_\alpha dp_z$.

If the conditions (10) are fulfilled for A waves, the ratio of their electric field components is

$$\Theta \equiv \frac{E_z^A}{E_x^A} \simeq \frac{k_x k_z c^2}{2\omega_p^2\langle\gamma(1+v_z/c)^2\rangle}. \tag{13}$$

III. GENERATION OF L WAVES AND THEIR EVOLUTION

We consider the model where L waves are generated by the two-stream instability that develops at the distance $r_i \sim 10^8$ cm from the neutron star surface because of overlap of the pair plasma clouds ejected from the pulsar [10]. Below, all calculations are done in the plasma frame where the mean vector velocity of the cloud particles is zero. We assume that in the regions where the plasma clouds overlap the densities of the slow and fast particles are the same (for the development of two-stream instability for an arbitrary distribution function of plasma particles, see [10]). In this case the Lorentz factor of the plasma frame in the pulsar frame is $\gamma_p \simeq (\gamma_{\max}\gamma_{\min})^{1/2} \simeq 10^2$.

The two-stream instability starts developing when the cloud overlap is very slight, and in the overlapping regions the momentum spreads of the slow and fast particles are small [10]; i.e., the distribution function roughly is $F_\alpha \simeq [\delta(p_z - p_0) + \delta(p_z + p_0)]$, where $p_0 = \gamma_0 m_e c$ and $\gamma_0 \simeq (1/2) \times (\gamma_{\max}/\gamma_{\min})^{1/2} \simeq 5$. For L waves with $k_x = 0$ the dispersion equation $\varepsilon_{zz} = 0$ provides the maximum growth rate $\Gamma_{\max} \simeq \omega_p/(\sqrt{2}\gamma_0^{3/2})$ which is achieved at $k_{z,opt} = \sqrt{6}\omega_p/(2V_0\gamma_0^{3/2})$, where $V_0 \simeq c$ is the particle velocity corresponding to the momentum p_0. Obliquely propagating L waves ($k_x \neq 0$) are also generated by the two-stream instability. For such waves with $k_x \lesssim k_z$ the growth rate is $\sim \Gamma_{\max}$ [11]. The characteristic frequency of generated L waves is $\sim \omega_p \langle\gamma_0\rangle^{1/2} \simeq \omega_p \gamma_0^{1/2}$ [6,10].

When the amplitudes of generated L waves become large enough, linear approximation is not valid anymore and the lowest-order nonlinear process called quasilinear relaxation starts up [12]. This process may be described by Eq. (5), and its characteristic time is

$$\tau_{\rm QL} \simeq \Lambda \gamma_0^{3/2}/\omega_p, \tag{14}$$

where Λ is the Coulomb logarithm. The quasilinear relaxation results in the distribution function of particles having the shape of a plateau up to the Lorentz factor of $\sim \gamma_0$; i.e., F_α is $\sim \gamma_0^{-1}$ at $\gamma \lesssim \gamma_0$ and nearly zero at $\gamma > \gamma_0$. Further generation of A waves is cut off.

After the stage of quasilinear relaxation the induced scattering of L waves by plasma particles becomes the dominant nonlinear process [12]. The characteristic time of this process is

$$\tau_{\rm IS} \simeq \Lambda \frac{\gamma_0^{3/2}}{\omega_p}\frac{n_0\gamma_0 mc^2}{W_L}, \tag{15}$$

where W_L is the energy density of L waves. The induced scattering transfers the wave energy from the frequency region where L waves are generated ($\omega \sim \omega_p\langle\gamma\rangle^{1/2} \simeq \omega_p\gamma_0^{1/2}$ and $k_z \sim k_{z,opt}$) to the low-frequency region ($\omega \sim \omega_p\langle\gamma^{-3}\rangle^{1/2} \simeq \omega_p\gamma_0^{-1/2}$ and $k_z \sim \omega_p\sqrt{\langle\gamma\rangle}/c \simeq \omega_p\gamma_0^{1/2}/c$); i.e., the mean frequency of L waves decreases $\sim \gamma_0 \simeq 5$ times. Here, we used that $\langle\gamma^n\rangle \sim \gamma_0^n$ for $n > 0$ and $\langle\gamma^{-n}\rangle \sim \gamma_0^{-1}$ for $n \geq 1$.

IV. NONLINEAR DECAY $L \rightarrow L' + A$

The process of nonlinear conversion of L waves into A waves, $L \rightarrow L' + A$ (and any other three-waves process), is described by the second-order current. From Eqs. (4) and (8) this current can be written as

$$j_z^{(2)} = \sum_\alpha \frac{e^3\omega}{m}\frac{\partial}{\partial p_z}\int d\bar{k}'\ E_z(\bar{k}-\bar{k}')E_z(\bar{k}') \times \int dp_z \frac{\partial f_\alpha^{(0)}/\partial p_z}{\omega' - k_z' v_z}\frac{1}{\gamma^3(\omega - k_z v_z)^2}, \quad (16)$$

where $E_z = E_z^L + E_z^A$ in the general case when both L and A waves participate in the process.

The distribution functions of electrons and positrons created via electromagnetic processes are identical. In such a pair plasma all three-waves processes are absent. This is because the second-order current $\mathbf{j}^{(2)}$ is proportional to e^3 and the summation in (16) over electrons and positrons gives $j_z^{(2)}=0$. However, in the process of outflow of pair plasma along the curved magnetic field lines, the distribution functions of electrons and positrons are shifted with respect to each other to maintain the electric neutrality of the plasma [13]. This results in the mean Lorentz factors of electrons ($\bar{\gamma}_e$) and positrons ($\bar{\gamma}_p$) slightly differing, $\Delta\gamma = |\bar{\gamma}_e - \bar{\gamma}_p| \sim 1$.

For L waves shifted to the low-frequency region by induced scattering, there is no solution of the resonant conditions $\omega_L = \omega_{L_1} + \omega_{L_2}$ and $\mathbf{k}_L = \mathbf{k}_{L_1} + \mathbf{k}_{L_2}$. Therefore, the process $L \to L_1 + L_2$ is kinematically forbidden and we can omit all terms proportional to $E_z^L(\bar{k})E_z^L(\bar{k}-\bar{k}')$ in Eq. (16). Taking into account that $\omega_L \gg k_L c$ and $\omega_A \approx k_A c$ for low-frequency L waves and A waves, respectively, from Eqs. (13) and (16) we obtain

$$-i(\omega - \omega_L)E_z^L = \int d\bar{k}\ V_{L'A} E_z^L(\bar{k}-\bar{k}')E_x^A(\bar{k}'), \quad (17)$$

where

$$V_{L'A} \approx \frac{4\pi e^3 n_0^2 \Delta\gamma\Theta}{m^2 c\,\omega_p \omega_A \gamma_0^{1/2}} \quad (18)$$

is the matrix element of interaction that, together with the resonant conditions

$$\omega_L = \omega_{L'} + \omega_A \quad \text{and} \quad \mathbf{k}_L = \mathbf{k}_{L'} + \mathbf{k}_A, \quad (19)$$

totally determines the conversion process $L \to L' + A$.

To describe the frequency change in the process $L \to L' + A$, it is convenient to introduce the ration of the frequency of generated A waves to the mean frequency of the low-frequency L waves, $\Omega_A \equiv \omega_A/(\sqrt{2}\langle\gamma^{-3}\rangle^{1/2}\omega_p) \simeq \omega_A/(\sqrt{2}\gamma_0^{-1/2}\omega_p)$. The results of numerical solution of the resonant conditions (19) are shown in Fig. 1 where the dimensionless wave number is used, $K_{x,y}^L \equiv k_{x,y}^L c/(\sqrt{2\langle\gamma^{-3}\rangle}\omega_p)$. We can see that for the typical parameters the frequency of generated A waves is about 2 orders of magnitude less than the mean frequency of L waves. To clarify our numerical results presented in Fig. 1 we can find the approximate analytical solution of the resonant conditions (19) for obliquely propagating L waves, $\Omega_A \approx (K_x^L)^2(K_z^L)^2/2 \ll 1$ at $K_x^{L'}=0$. From this solution it follows that for rather small wave numbers of L waves the frequency of generated A waves is always much less than the characteristic frequency of L waves.

Using the standard technique of weak turbulence theory (e.g., [14]), the characteristic time scale of nonlinear decay $L \to L' + A$ for the waves with random phases may be written as

$$\tau_{NLA} \sim \frac{\omega_L^2}{W_L V_{L'A}^2 \omega_A}. \quad (20)$$

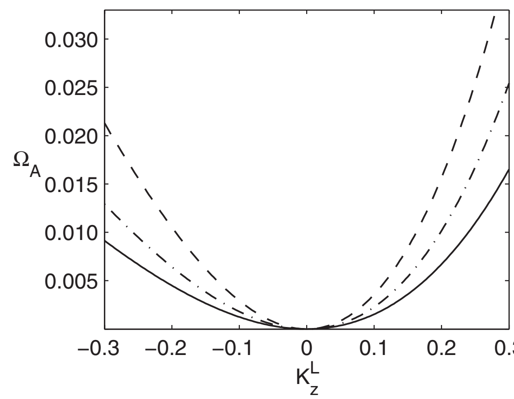

FIG. 1. Dimensionless frequency of A waves generated in the process $L \to L' + A$ as a function of z component of dimensionless wave number of L waves before their decay for different parameters: $K_x^L=0.5$, $K_x^{L'}=0$ (solid line), $K_x^L=0.7$, $K_x^{L'}=0$ (dashed line), and $K_x^L=0.7$, $K_x^{L'}=0.4$ (dash-dotted line).

V. NUMERICAL ESTIMATES AND DISCUSSION

If we assume that in the plasma frame the frequency of radio emission coincides with the mean frequency of L waves generated by the two-stream instability at the distance r_i, the characteristic frequency of the radio emission in the pulsar frame is

$$\nu_L \sim 10(B_{12} M \gamma_p \gamma_0 / P_{0.1})^{1/2}(R/r_i)^{3/2}\ \text{GHz}, \quad (21)$$

where B_{12} is the magnetic field at the neutron star surface in units of 10^{12} G, $P_{0.1}$ is the period of the pulsar rotation in units of 0.1 s, $R \simeq 10^6$ cm is the neutron star radius, and M is a so-called multiplicity factor, which is equal to the ratio of the pair plasma density to the density of primary particles [7]. In early papers where only the curvature radiation of primary particles was considered as the mechanism of generation of γ rays, the pair multiplicity is rather high, $M \simeq 10^3 - 10^4$ [5,15]. Later, it was shown that generation of γ rays by inverse Compton scattering is essential near at least some pulsars [16]. In this case the value of M may be as small as $\sim 1-10$. For plausible parameters (e.g., $\gamma_p \sim 10^2$, $\gamma_0 \sim 5$, $M \sim 10-10^3$, $B_{12} \sim 1$, $P_{0.1} \sim 1$, and $r_i/R = 50$) Eq. (21) yields $\nu_L \sim 2-20$ GHz, which is about an order of magnitude larger than the typical frequency of pulsar radio emission.

In this paper we discussed the following sequence of processes in the magnetospheres of pulsars: (a) the development of two-stream instability in collisions of plasma clouds and generation of L waves, (b) quasilinear relaxation of two-stream instability, (c) induced scattering of generated L waves by plasma particles and decrease of the mean frequency of L waves, and (d) nonlinear conversion of L waves into low-frequency A waves. For plausible parameters of the pulsar plasma defined above and for $K^L_{x,z}=0.3$ and $W_L \sim 0.1 n_0 \gamma_0 mc^2$, the characteristic times of these processes in the plasma frame are $\tau_i = (\Gamma_{\max})^{-1} \sim 10^{-8}$ s, $\tau_{QL} \sim 10^{-7}$ s, $\tau_{IS} \sim 10^{-6}$ s, and $\tau_{NLA} \sim 10^{-6}$ s, respectively. These times are at least an order of magnitude smaller than the characteristic time of the plasma outflow, $\tau_{out} \simeq r_i/(c\gamma_p) \simeq 2\times 10^{-5}$ s. Therefore, all these processes have enough time to be developed in the magnetospheres of pulsars. We have shown that the frequency of A waves is a few 10 times smaller than the frequency of generated L waves and may be consistent with the observational data on the radio emission of pulsars.

Gedalin, Gruman, and Melrose [17] proposed another mechanism of pulsar radio emission that also involves the two-stream instability and solves successfully the high-frequency problem at least for the main part of known pulsars. In this mechanism obliquely propagating electromagnetic waves are directly generated by the instability. At present, the mechanism by Gedalin *et al.* has an advantage over our mechanism because electromagnetic waves generated by their mechanism can escape from the magnetospheres of pulsars (provided cyclotron absorption is unimportant) while it was argued (e.g., [18]) that A waves are subject to strong Landau damping and cannot be observed far from the pulsar (but see [19]). However, in the simple model used in [18] it was assumed that the magnetosphere plasma is stationary and that the plasma density declines in proportion to r^{-3} without other variations. These assumptions are invalid in our case where the pair plasma outflow is nonstationary and strongly nonuniform across and along the magnetic field lines [5,6]. At the edges of the plasma clouds the density falls down sharply and significantly, and A waves may be converted into nondamping waves that escape as radio emission. Besides, in the spaces between the plasma clouds the density and Landau damping of A waves may be small. A study of these effects is beyond the framework of this paper and will be addressed elsewhere.

In turn, our mechanism has an advantage over the mechanism by Gedalin *et al.* for pulsars with the maximum of the radio spectra at very low frequencies ($<10^2$ MHz) because the mean frequency of radio emission in our mechanism may be an order less than the same in the mechanism by Gedalin *et al.*

ACKNOWLEDGMENTS

This work was supported in part by Georgian NSF Grant No. 06-58-4-320, INTAS Grant No. 06-1000017-9258, and the Israel Science Foundation of the Israel Academy of Sciences and Humanities.

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